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HYDRODYNAMIC COEFFICIENT CALCULATION USING DOUGLAS POTENTIAL FLOW COMPUTER PROGRAM
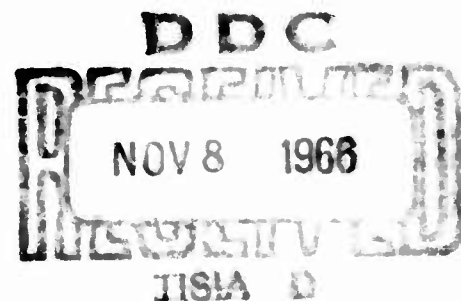
by

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ABSTRACT. A new method is presented for calculating the hydrodynamic coefficients of axisymmetric bodies. The method utilizes an extended version of the Douglas potential flow computer program for axisymmetric bodies to calculate the potential and velocity distribution along the body required for the determination of the forces and moment acting on the body. Based on a comparison with experiment limited to one body shape, the method yields values of the hydrodynamic coefficients for blunt- or flat-based bodies which are adequate for most engineering work.

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FOREWORD

This report presents a new method, using the Douglas potential flow computer program, for calculating the hydrodynamic coefficients of axisymmetric bodies. The development given here represents the first phase of this work, which is restricted to blunt- or flat-based bodies. The next phase, involving an extension to streamlined bodies, will be presented in a future report.

The work was undertaken to determine a general analytic method for calculating the hydrodynamic coefficients of axisymmetric bodies. It was done from April 1964 - July 1965 under Bureau of Naval Weapons Special Projects Task Assignment 24401, with P. R. Faurot as BuWeps project engineer. This report represents the considered opinions of the Propulsion Division.

CONTENTS

NOMENCLATURE

B	Equation of body surface
F_A	Axial force on body
F_T	Transverse force on body
f_u, f_w, f_q	Functions describing variation of v_s along body
h_u, h_w, h_q	Functions describing variation of ϕ along body
i, j, k	Unit vectors in the x, y, z directions, respectively
l	Body length
M	Moment on body
$M'_{\dot{w}}$	Nondimensional derivative of moment with respect to $\dot{w}$
$M'_{\dot{q}}$	Nondimensional derivative of moment with respect to $\dot{q}$
M'_w	Nondimensional derivative of moment with respect to w
M'_q	Nondimensional derivative of moment with respect to q
N	Unit vector normal to body surface
p	Fluid pressure
q	Angular velocity of body
R	Body radius
r	Radial coordinate
s, n, θ	Body fixed orthogonal curvilinear coordinates
t	Time
u	Axial velocity of body
V	Fluid velocity
V_B	Velocity of body surface
v_s, v_n, v_θ	Fluid velocity components in the s, n, θ directions, respectively
w	Transverse velocity of body
X, Y, Z	Inertial rectangular coordinates
X_o	X coordinate of body CG
$X'_{\dot{u}}$	Nondimensional derivative of axial force with respect to $\dot{u}$
x, y, z	Body fixed rectangular coordinates
Z_o	Z coordinate of body CG

$Z'_{\dot{w}}$ Nondimensional derivative of transverse force with respect to $\dot{w}$

$Z'_{\dot{q}}$ Nondimensional derivative of transverse force with respect to $\dot{q}$

Z'_{w} Nondimensional derivative of transverse force with respect to w

Z'_{q} Nondimensional derivative of transverse force with respect to q

β Angle that a body-surface tangent makes with the axis

γ Angle the body axis makes with respect to the horizontal

ρ Mass density of fluid

ϕ Potential generated by body

SUBSCRIPTS

u Due to axial velocity

w Due to transverse velocity

q Due to angular velocity

s In direction of increasing s

n In direction of increasing n

θ In direction of increasing θ

INTRODUCTION

Traditionally, there have been only two methods for estimating the hydrodynamic coefficients of axisymmetric bodies. For bodies having relatively large length-to-diameter ratios and blunt or truncated bases, slender body theory can be used. For bodies having a streamlined afterbody, the semiempirical methods of Ref. 1 and 2 can be used, provided the shape does not deviate too much from that of a typical torpedo. However, for blunt-based bodies having small length-to-diameter ratios, or for streamlined bodies having shapes that fall well outside the empirical data of Ref. 1 and 2, there exists no satisfactory method for estimating hydrodynamic coefficients. To fill this void and, hopefully, to improve upon the two methods mentioned above, the work presented here was undertaken.

For bodies moving with a large axial velocity, small angle of attack, and small pitch rate, so that no large amount of vortex shedding occurs, it is reasonable to look upon the flow around the body as being made up of two regions: (1) a region near the body where viscosity and turbulence play an important part, i. e., the boundary layer; and (2) a region outside the boundary layer where the flow is strictly potential. Bodies having blunt or truncated bases normally do not develop thick boundary layers; hence they may be treated as though the flow around them were purely potential. Of course, the fact that the flow separates from the body at the base must be properly accounted for in the treatment. Bodies having streamlined afterbodies normally develop thick boundary layers there, so that any treatment of them must analyze both the boundary layer and potential flow. Although the ultimate purpose of this work is to facilitate the handling of the case where a thick boundary layer is present, the analysis presented in this report is restricted to blunt-based bodies where the boundary layer may be neglected. It is planned to combine a technique[1] for analyzing the boundary layer with an analysis similar to that given here to handle the case of a streamlined afterbody. This work will be presented in a future report.

In seeking a method to calculate the potential flow about an axisymmetric body of arbitrary shape, it soon became apparent that the method developed at the Douglas Aircraft Co. (Ref. 4 and 5) had no close rival. Not only was the method essentially exact, but a computer program was available. It was therefore decided to build the method for calculating hydrodynamic coefficients around the Douglas potential flow solution.

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[1] The first phase of this boundary layer work is presented in Ref. 3.

EXTENSION OF THE DOUGLAS PROGRAM

When the idea of using the Douglas potential flow computer program to calculate the hydrodynamic coefficients of axisymmetric bodies was conceived, it was possible to use the program to obtain values of the fluid velocity at the body surface for a stream flow parallel to the axis and a uniform crossflow. Since this was not enough information to calculate a complete set of coefficients, a contract was let to the Douglas Aircraft Co. to extend its theoretical work and include in the computer program the following additional capabilities.

1. Solution for nonuniform crossflow
2. Calculation of the perturbation potential at the body surface for axisymmetric flows and crossflows
3. Calculation of the perturbation velocity at the body surface for axisymmetric flows and crossflows

With these additions to the program, all quantities required for the determination of a complete set of coefficients were available. A discussion of theoretical development work and extentions to the computer program is given in Ref. 6.

The term "nonuniform crossflow" used above is admittedly not explicit. More precisely, this condition may be defined as any flow where the potential generated by the body satisfies a boundary condition of the type

$$\left(\frac{\partial \phi}{\partial n}\right)_{n=0} = (v_n)_{n=0} = f(s) \cos \theta$$

The terms "perturbation potential" and "perturbation velocity" are the potential and velocity arising from the presence of a body in a moving mass of fluid. They are, of course, equal to the potential and velocity generated by a body moving in still fluid, provided the relative motion of the body and the fluid at infinity is equal.

PHYSICAL ARRANGEMENT

The physical arrangement considered is shown in Fig. 1. An axisymmetric body is in planar motion in unlimited fluid with an axial velocity u, a transveise velocity w, and an angular velocity q. The plane of motion is the X-Z plane of the inertial X, Y, Z coordinate system. Two coordinate systems are fixed in the body: a rectangular x, y, z system originating at the center of gravity of the body, and an orthogonal curvilinear s, n, θ system with the origin of the s coordinate at the body nose. An angle γ is used to define the orientation of the body axis relative to the X axis, and coordinates X_o and Z_o are used to locate the position of the body CG in the X-Z plane.
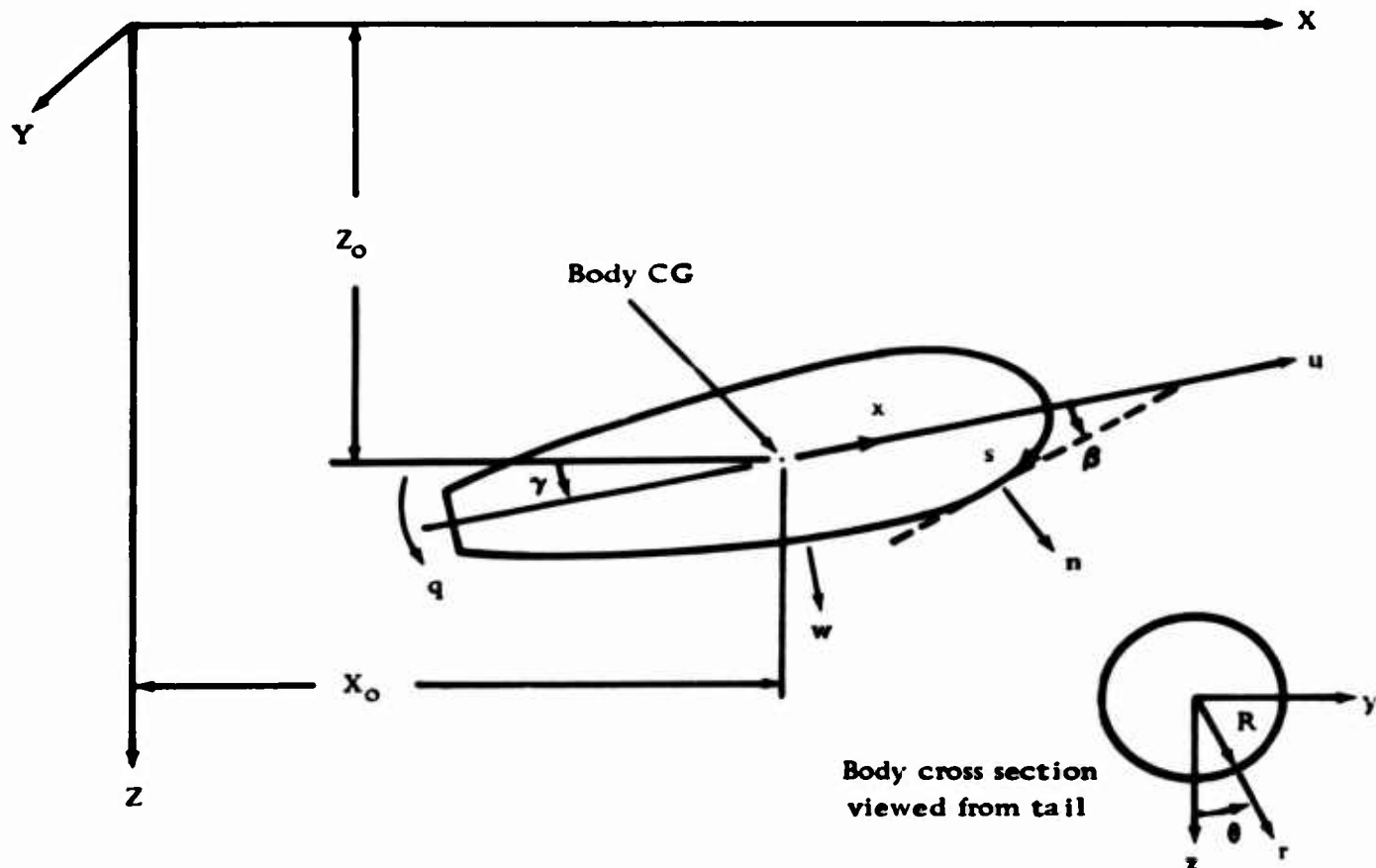
FIG. 1. Physical Arrangement.

BOUNDARY CONDITIONS

The required condition at the body surface is one in which no fluid penetrates the surface and no gaps occur between the fluid and the surface. Hence the fluid velocity component normal to the surface must equal the velocity of the surface normal to itself. The following equation states this requirement.

$$(v_n)_{n=0} = \vec{V}_B \cdot \vec{N} \tag{1}$$

V_B is the velocity of the body surface and N is the unit normal vector to the body surface. N is given by the relationship

$$\vec{N} = \frac{\text{grad } B}{\sqrt{\left(\frac{\partial B}{\partial x}\right)^2 + \left(\frac{\partial B}{\partial y}\right)^2 + \left(\frac{\partial B}{\partial z}\right)^2}} = \frac{\frac{\partial B}{\partial x} i + \frac{\partial B}{\partial y} j + \frac{\partial B}{\partial z} k}{\sqrt{\left(\frac{\partial B}{\partial x}\right)^2 + \left(\frac{\partial B}{\partial y}\right)^2 + \left(\frac{\partial B}{\partial z}\right)^2}} \tag{2}$$

where B (the equation of the body surface) = $f(x, y, z) = 0$. Since the body is axisymmetric, B may be expressed[2] as

$$B = \sqrt{y^2 + z^2} - R(x) = 0$$

from which there is obtained

$$\frac{\partial B}{\partial x} = \frac{\partial B}{\partial R} \cdot \frac{\partial R}{\partial x} = -\frac{dR}{dx}$$

$$\frac{\partial B}{\partial y} = \frac{y}{\sqrt{y^2 + z^2}}$$

$$\frac{\partial B}{\partial z} = \frac{z}{\sqrt{y^2 + z^2}} \tag{3}$$

Substituting Eq. 3 into Eq. 2 and making use of the fact (see Fig. 1) that at the body surface $dR/dx = -\tan\beta$, $y = R \sin\theta$, and $z = R \cos\theta$, there results

$$\vec{N} = \sin\beta i + \sin\theta \cos\beta j + \cos\theta \cos\beta k \tag{4}$$

Using Fig. 1, the velocity of the body surface may be written directly.

For axial motion u: $\qquad \vec{V}_B = ui + 0j + 0k$

For transverse motion w: $\qquad \vec{V}_B = 0i + 0j + wk$

For rotation q: $\qquad \vec{V}_B = qR \cos\theta i + 0j - qxk \tag{5}$

Substituting Eq. 4 and 5 into Eq. 1, the boundary condition equation is obtained.

For axial motion u: $\qquad (v_{un})_{n=0} = u \sin\beta$

For transverse motion w: $\qquad (v_{wn})_{n=0} = w \cos\beta \cos\theta$

For rotation q: $\qquad (v_{qn})_{n=0} = q(R \sin\beta - x \cos\beta) \cos\theta \tag{6}$

The subscripts u, w, and q are introduced here to mean resulting from axial motion, transverse motion, and rotation, respectively.

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[2] Writing the expression for B in this manner implies that R is a single-valued function of x. However, it is easily shown that the resulting expression, Eq. 4, is completely general and applies equally well to bodies where R is a multivalued function of x, such as flat-nosed bodies.

FLUID POTENTIAL

The potential generated by the body will be made up of three parts arising from the three separate motions of the body: u, w, and q

$$\phi = \phi_u + \phi_w + \phi_q \tag{7}$$

Since the potential generated by an axisymmetric body moving along its axis of symmetry is naturally axisymmetric, the most general expression for ϕ_u is

$$\phi_u = uF_u(s, n) \tag{8}$$

In Ref. 5, Hess shows that the potential generated by an axisymmetric body satisfying a boundary condition of the type

$$(v_u)_{n=0} = f(s) \cos \theta$$

has the simple form

$$\phi = f(s, n) \cos \theta$$

From Eq. 6, therefore, it is apparent that the most general expressions for ϕ_w and ϕ_q are

$$\phi_w = wF_w(s, n) \cos \theta$$

$$\phi_q = qF_q(s, n) \cos \theta \tag{9}$$

PRESSURE AT THE BODY SURFACE

BERNOULLI EQUATION

Since the body is not in steady motion relative to any possible choice of inertial coordinate system, the nonsteady Bernoulli equation must be used to determine the pressure at the body surface. Assuming the convention $\vec{V} = \text{grad}\,\phi$ where V is the fluid velocity, the pressure at the body surface is given by

$$P_{n=0} = -\rho \left(\frac{\partial \phi}{\partial t} \right)_{n=0} - \tfrac{1}{2} \rho (V^2)_{n=0} \tag{10}$$

EVALUATION OF $(\partial\phi/\partial t)_{n=0}$

The $\partial\phi/\partial t$ in the nonsteady Bernoulli equation is the time rate of change of the potential at a point fixed in inertial space; i.e.,

$$\frac{\partial\phi(X, Y, Z, t)}{\partial t}$$

However, ϕ is known as a function of s, n, θ, u, w, and q where

$$s = f(x, r)$$

$$n = f(x, r)$$

$$x = f(X, Y, Z, t)$$

$$r = f(X, Y, Z, t)$$

$$\theta = f(X, Y, Z, t)$$

$$u = f(t)$$

$$w = f(t)$$

$$q = f(t)$$

Therefore, the chain rule must be used. Making use of Eq. 7, there is obtained

$$\left(\frac{\partial\phi}{\partial t}\right)_{n=0} = \left[\left(\frac{\partial\phi_u}{\partial s}\right)_{n=0} + \left(\frac{\partial\phi_w}{\partial s}\right)_{n=0} + \left(\frac{\partial\phi_q}{\partial s}\right)_{n=0}\right]\left[\left(\frac{\partial s}{\partial x}\right)_{n=0}\left(\frac{\partial x}{\partial t}\right)_{r=R} + \left(\frac{\partial s}{\partial r}\right)_{n=0}\left(\frac{\partial r}{\partial t}\right)_{r=R}\right]$$

$$+ \left[\left(\frac{\partial\phi_u}{\partial n}\right)_{n=0} + \left(\frac{\partial\phi_w}{\partial n}\right)_{n=0} + \left(\frac{\partial\phi_q}{\partial n}\right)_{n=0}\right]\left[\left(\frac{\partial n}{\partial x}\right)_{n=0}\left(\frac{\partial x}{\partial t}\right)_{r=R} + \left(\frac{\partial n}{\partial r}\right)_{n=0}\left(\frac{\partial r}{\partial t}\right)_{r=R}\right]$$

$$+ \left[\left(\frac{\partial\phi_u}{\partial\theta}\right)_{n=0} + \left(\frac{\partial\phi_w}{\partial\theta}\right)_{n=0} + \left(\frac{\partial\phi_q}{\partial\theta}\right)_{n=0}\right]\left(\frac{\partial\theta}{\partial t}\right)_{r=R}$$

$$+ \left(\frac{\partial\phi_u}{\partial u}\right)_{n=0}\dot{u} + \left(\frac{\partial\phi_w}{\partial w}\right)_{n=0}\dot{w} + \left(\frac{\partial\phi_q}{\partial q}\right)_{n=0}\dot{q} \qquad (11)$$

where the dot over a symbol indicates the derivative with respect to time; i.e., $\dot{u} = du/dt$ etc.

Equations 6, 8, and 9 allow the following relationships to be written.

$$\left(\frac{\partial \phi_u}{\partial s}\right)_{n=0} = (v_{us})_{n=0} = u\left(\frac{\partial F_u}{\partial s}\right)_{n=0} = uf_u(s)$$

$$\left(\frac{\partial \phi_w}{\partial s}\right)_{n=0} = (v_{ws})_{n=0} = w\left(\frac{\partial F_w}{\partial s}\right)_{n=0} \cos\theta \approx wf_w(s)\cos\theta$$

$$\left(\frac{\partial \phi_q}{\partial s}\right)_{n=0} = (v_{qs})_{n=0} = q\left(\frac{\partial F_q}{\partial s}\right)_{n=0} \cos\theta = qf_q(s)\cos\theta$$

$$\left(\frac{\partial \phi_u}{\partial n}\right)_{n=0} = (v_{un})_{n=0} = u\sin\beta$$

$$\left(\frac{\partial \phi_w}{\partial n}\right)_{n=0} = (v_{wn})_{n=0} = w\cos\beta\cos\theta$$

$$\left(\frac{\partial \phi_q}{\partial n}\right)_{n=0} = (v_{qn})_{n=0} = q(R\sin\beta - x\cos\beta)\cos\theta$$

$$\left(\frac{\partial \phi_u}{\partial\theta}\right)_{n=0} = 0$$

$$\left(\frac{\partial \phi_w}{\partial\theta}\right)_{n=0} = R(v_{w\theta})_{n=0} = -w(F_w)_{n=0}\sin\theta = -wh_w(s)\sin\theta$$

$$\left(\frac{\partial \phi_q}{\partial\theta}\right)_{n=0} = R(v_{q\theta})_{n=0} = -q(F_q)_{n=0}\sin\theta = -qh_q(s)\sin\theta$$

$$\left(\frac{\partial \phi_u}{\partial u}\right)_{n=0} = (F_u)_{n=0} = h_u(s)$$

$$\left(\frac{\partial \phi_w}{\partial w}\right)_{n=0} = (F_w)_{n=0}\cos\theta = h_w(s)\cos\theta$$

$$\left(\frac{\partial \phi_q}{\partial q}\right)_{n=0} = (F_q)_{n=0}\cos\theta = h_q(s)\cos\theta \tag{12}$$

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From Fig. 1, it is seen that s and n are functions of x and r through the relations

$$x = x_{nose} - \int_0^s \cos\beta \, ds + n \sin\beta \tag{13}$$

$$r = \int_0^s \sin\beta \, ds + n \cos\beta \tag{14}$$

where $\beta = f(s)$. Taking the partial derivative of Eq. 13 and 14 with respect to x and solving simultaneously for $\partial n/\partial x$ and $\partial s/\partial x$, and taking the partial derivative with respect to r and solving simultaneously for $\partial n/\partial r$ and $\partial s/\partial r$, there is obtained

$$\left(\frac{\partial n}{\partial x}\right)_{n=0} = \sin\beta$$

$$\left(\frac{\partial s}{\partial x}\right)_{n=0} = -\cos\beta$$

$$\left(\frac{\partial n}{\partial r}\right)_{n=0} = \cos\beta$$

$$\left(\frac{\partial s}{\partial r}\right)_{n=0} = \sin\beta \tag{15}$$

From Fig. 1, the time rates of change of (1) the coordinates of the body center of gravity and (2) the angle between the body axis and the horizontal are

$$\frac{dX_o}{dt} = u \cos\gamma + w \sin\gamma$$

$$\frac{dZ_o}{dt} = -u \sin\gamma + w \cos\gamma$$

$$\frac{d\gamma}{dt} = q \tag{16}$$

Also from Fig. 1, the coordinates of a point in the inertial X, Y, Z system in terms of the coordinate system in the body are

$$X = X_o + x \cos \gamma + r \cos \theta \sin \gamma$$

$$Y = r \sin \theta$$

$$Z = Z_o - x \sin \gamma + r \cos \theta \cos \gamma \qquad (17)$$

Taking the partial derivative of Eq. 17 with respect to time, there results

$$0 = \frac{dX_o}{dt} - x \sin \gamma \frac{d\gamma}{dt} + \cos \gamma \frac{\partial x}{\partial t} + r \cos \theta \cos \gamma \frac{d\gamma}{dt}$$

$$+ \sin \gamma \left(-r \sin \theta \frac{\partial \theta}{\partial t} + \cos \theta \frac{\partial r}{\partial t} \right)$$

$$0 = r \cos \theta \frac{\partial \theta}{\partial t} + \sin \theta \frac{\partial r}{\partial t}$$

$$0 = \frac{dZ_o}{dt} - x \cos \gamma \frac{d\gamma}{dt} - \sin \gamma \frac{\partial x}{\partial t} - r \cos \theta \sin \gamma \frac{d\gamma}{dt}$$

$$+ \cos \gamma \left(-r \sin \theta \frac{\partial \theta}{\partial t} + \cos \theta \frac{\partial r}{\partial t} \right) \qquad (18)$$

Substituting Eq. 16 into Eq. 18 and solving simultaneously for $\partial x/\partial t$, $\partial r/\partial t$, and $\partial \theta/\partial t$, there is obtained

$$\left(\frac{\partial x}{\partial t} \right)_{r=R} = -u - qR \cos \theta$$

$$\left(\frac{\partial r}{\partial t} \right)_{r=R} = -(w - qx) \cos \theta$$

$$\left(\frac{\partial \theta}{\partial t} \right)_{r=R} = (w - qx) \frac{\sin \theta}{R} \qquad (19)$$

Substituting Eq. 12, 15, and 19 into Eq. 11, the expression for the $(\partial \phi/\partial t)_{n=0}$ is as follows:

$$\left(\frac{\partial \phi}{\partial t}\right)_{n=0} = (uf_u + wf_w \cos\theta + qf_q \cos\theta)(u\cos\beta + qR\cos\beta\cos\theta$$

$$- w\sin\beta\cos\theta + qx\sin\beta\cos\theta) + (u\sin\beta + w\cos\beta\cos\theta$$

$$+ qR\sin\beta\cos\theta - qx\cos\beta\cos\theta)(-u\sin\beta - qR\sin\beta\cos\theta$$

$$- w\cos\beta\cos\theta + qx\cos\beta\cos\theta) + (-wh_w\sin\theta - qh_q\sin\theta)$$

$$\left(w\frac{\sin\theta}{R} - qx\frac{\sin\theta}{R}\right) + \dot{u}h_u + \dot{w}h_w\cos\theta + \dot{q}h_q\cos\theta \quad (20)$$

EVALUATION OF $(V^2)_{n=0}$

The square of the fluid velocity will be equal to the sum of the squares of the three orthogonal components v_s, v_n, and v_θ.

$$(V^2)_{n=0} = [(v_{us})_{n=0} + (v_{ws})_{n=0} + (v_{qs})_{n=0}]^2$$

$$+ [(v_{un})_{n=0} + (v_{wn})_{n=0} + (v_{qn})_{n=0}]^2$$

$$+ [(v_{u\theta})_{n=0} + (v_{w\theta})_{n=0} + (v_q)_{n=0}]^2 \quad (21)$$

Substituting Eq. 12 into Eq. 21, the expression for $(V^2)_{n=0}$ is

$$(V^2)_{n=0} = (uf_u + wf_w \cos\theta + qf_q \cos\theta)^2$$

$$+ [u\sin\beta + w\cos\beta\cos\theta + q(R\sin\beta - x\cos\beta)\cos\theta]^2$$

$$+ \left(-wh_w\frac{\sin\theta}{R} - qh_q\frac{\sin\theta}{R}\right)^2 \quad (22)$$

TRANSVERSE FORCE, AXIAL FORCE, AND MOMENT ON BODY

Utilizing Fig. 1, the expressions for the transverse force F_T, axial force F_A, and moment M on an elemental surface distance of the body, ds, can be written directly. The transverse force is determined by integrating, around the circumference of the body, the component of the pressure force in the F_T direction.

$$\frac{dF_T}{ds} = -R\cos\beta \int_0^{2\pi} P_{n=0}\cos\theta\, d\theta \quad (23)$$

The axial force is determined by integrating, around the body, the component of the pressure force in the F_A direction.

$$\frac{dF_A}{ds} = -R \sin\beta \int_0^{2\pi} p_{n=0} \, d\theta \tag{24}$$

The moment is determined by integrating around the body the contributions arising from (1) the product of the pressure force transverse component and the lever arm x, and (2) the product of the axial component of the pressure force and the lever arm R cos θ.

$$\frac{dM}{ds} = \underbrace{Rx \cos\beta \int_0^{2\pi} p_{n=0} \cos\theta \, d\theta}_{\substack{\text{contribution from} \\ \text{transverse pressure} \\ \text{force}}} - \underbrace{R^2 \sin\beta \int_0^{2\pi} p_{n=0} \cos\theta \, d\theta}_{\substack{\text{contribution from} \\ \text{axial pressure} \\ \text{force}}} \tag{25}$$

Substituting Eq. 20 and 22 into Eq. 10 yields the expression for the pressure on the body surface, $p_{n=0}$. Putting this expression into Eq. 23, 24, and 25, carrying out the indicated multiplications, and integrating with respect to θ around the body, and with respect to s along the body from the nose (s = 0) to the tail (s = s_{TAIL}), there results

$$F_T = \rho uw\pi \int_0^{s_{TAIL}} R \cos\beta(-\sin\beta \cos\beta - f_u \sin\beta + f_w \cos\beta + f_u f_w)ds$$

$$+ \rho uq\pi \int_0^{s_{TAIL}} R \cos\beta(-R \sin^2\beta + x \sin\beta \cos\beta + Rf_u \cos\beta$$

$$+ xf_u \sin\beta + f_q \cos\beta + f_u f_q)ds + \rho\dot{w}\pi \int_0^{s_{TAIL}} R \cos\beta h_w ds$$

$$+ \rho\dot{q}\pi \int_0^{s_{TAIL}} R \cos\beta h_q ds$$

$$F_A \text{ (apparent mass)} = \rho\dot{u}2\pi \int_0^{s_{TAIL}} R \sin\beta h_u ds$$

$$M = \rho uw\pi \int_0^{s_{TAIL}} R(R \sin\beta - x \cos\beta)(-\sin\beta \cos\beta - f_u \sin\beta$$

$$+ f_w \cos\beta + f_u f_w)ds + \rho uq\pi \int_0^{s_{TAIL}} R(R \sin\beta - x \cos\beta)(-R \sin^2\beta$$

$$+ x \sin\beta \cos\beta + Rf_u \cos\beta + xf_u \sin\beta + f_q \cos\beta + f_u f_q)ds$$

$$+ \rho\dot{w}\pi \int_0^{s_{TAIL}} R(R \sin\beta - x \cos\beta)h_w ds$$

$$+ \rho\dot{q}\pi \int_0^{s_{TAIL}} R(R \sin\beta - x \cos\beta)h_q ds \tag{26}$$

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Since the only potential axial force of practical significance is the axial apparent mass, all the terms in $p_{n=0}$, except the term containing $\dot{u}$, were dropped when determing F_A.

HYDRODYNAMIC COEFFICIENTS

The expressions for F_T, F_A, and M in terms of the SNAME (see Ref. 7) hydrodynamic nomenclature are

$$F_T = Z'_{\dot{w}} \tfrac{1}{2}\rho\ell^3 \dot{w} + Z'_{\dot{q}} \tfrac{1}{2}\rho\ell^4 \dot{q} + Z'_w \tfrac{1}{2}\rho\ell^2 uw + Z'_q \tfrac{1}{2}\rho\ell^3 uq$$

$$F_A \text{ (apparent mass)} = X'_{\dot{u}} \tfrac{1}{2}\rho\ell^3 \dot{u}$$

$$M = M'_{\dot{w}} \tfrac{1}{2}\rho\ell^4 \dot{w} + M'_{\dot{q}} \tfrac{1}{2}\rho\ell^5 \dot{q} + M'_w \tfrac{1}{2}\rho\ell^3 uw + M'_q \tfrac{1}{2}\rho\ell^4 uq \tag{27}$$

Equating the expressions for F_T, F_A, and M given by Eq. 26 and 27, and solving for the hydrodynamic coefficients, the following is obtained:

$$Z'_{\dot{w}} = \frac{\pi \int_0^{S_{TAIL}} R \cos\beta h_w \, ds}{\tfrac{1}{2}\ell^3}$$

$$Z'_{\dot{q}} = \frac{\pi \int_0^{S_{TAIL}} R \cos\beta h_q \, ds}{\tfrac{1}{2}\ell^4}$$

$$Z'_w = \frac{\pi \int_0^{S_{TAIL}} R \cos\beta(-\sin\beta \cos\beta - f_u \sin\beta + f_w \cos\beta + f_u f_w)ds}{\tfrac{1}{2}\ell^2}$$

$$Z'_q = \frac{\pi \int_0^{S_{TAIL}} R \cos\beta(-R \sin^2\beta + x \sin\beta \cos\beta + Rf_u \cos\beta + xf_u \sin\beta + f_q \cos\beta + f_u f_q)ds}{\tfrac{1}{2}\ell^3}$$

$$M'_{\dot{w}} = \frac{\pi \int_0^{S_{TAIL}} R(R \sin\beta - x \cos\beta)h_w \, ds}{\tfrac{1}{2}\ell^4}$$

$$M'_{\dot{q}} = \frac{\pi \int_0^{S_{TAIL}} R(R \sin\beta - x \cos\beta)h_q \, ds}{\tfrac{1}{2}\ell^5}$$

$$M'_w = \frac{\pi \int_0^{S_{TAIL}} R(R \sin\beta - x \cos\beta)(-\sin\beta \cos\beta - f_u \sin\beta + f_w \cos\beta + f_u f_w)ds}{\tfrac{1}{2}\ell^3}$$

$$M'_q = \frac{\pi \int_0^{s_{TAIL}} R(R \sin\beta - x\cos\beta)(-R\sin^2\beta + x\sin\beta\cos\beta + Rf_u\cos\beta + xf_u\sin\beta + f_q\cos\beta + f_uf_q)ds}{\frac{1}{2}\ell^4}$$

$$X'_u = \frac{2\pi \int_0^{s_{TAIL}} R\sin\beta h_u\, ds}{\frac{1}{2}\ell^3} \tag{28}$$

EVALUATION OF INTEGRALS

The integrals in Eq. 28 that express the hydrodynamic coefficients are functions of the body geometry and the functions $f_u(s)$, $f_w(s)$, $f_q(s)$, $h_u(s)$, $h_w(s)$, and $h_q(s)$. From Eq. 8, 9, and 12 it is seen that

$$(v_{us})_{n=0} = uf_u(s)$$

$$(v_{ws})_{n=0} = wf_w(s)\cos\theta$$

$$(v_{qs})_{n=0} = qf_q(s)\cos\theta$$

$$(\phi_u)_{n=0} = u(F_u)_{n=0} = uh_u(s)$$

$$(\phi_w)_{n=0} = w(F_w)_{n=0}\cos\theta = wh_w(s)\cos\theta$$

$$(\phi_q)_{n=0} = q(F_q)_{n=0}\cos\theta = qh_q(s)\cos\theta$$

Hence the functions f_u, f_w, and f_q describe the variation of the fluid-velocity component, in the s direction at the body surface and in the meridian plane $\theta = 0$, for unit values of u, w, and q. Further, the functions h_u, h_w, and h_q describe the variation of the potential at the body surface in the meridian plane $\theta = 0$ for unit values of u, w, and q. As discussed earlier, these functions are given by the extended Douglas program. They are obtained from the present[3] version of this program in the following manner. To obtain f_u and h_u, the following items are checked on the control cards: (1) surface of revolution, (2) perturbations only, and (3) potential computed. In the outputs, $T = f_u$ and PHI $= h_u$. To obtain f_w and h_w, the items checked on the control cards are (1) crossflow, (2) perturbations only, and (3) potential computed. In the outputs, $T_2 = -f_w$ and PHI $= -h_w$. To obtain f_q and h_q, the items checked on the control cards are (1) nonuniform onset flow (computed), (2) perturbations only, and (3) potential computed. In the outputs, $T_2 = f_q$ and PHI $= h_q$.

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[3] A new program for calculating the potential flow about axisymmetric bodies is currently being developed at the Douglas Aircraft Co. This program is written in FORTRAN IV, in contrast to the present machine-language version that operates in the SAMSON system.

A computer program was written at this Station to perform the integrations indicated in Eq. 28. Since the values of f_u, f_w, f_q and h_u, h_w, h_q obtained from the Douglas program will not normally occur at equally spaced values of s, values of the integrands are calculated at the points where f_u, f_w, f_q and h_u, h_w, h_q are known from the Douglas program solution. Values of the integrands at equally spaced values of s are then determined by three-point Lagrange interpolation, and the integrals are evaluated using Simpson's rule. An increasingly larger number of points is used in the Simpson-rule integration, until the values of the hydrodynamic coefficients reach an essentially constant value.

A COMPARISON WITH EXPERIMENT

Since the analysis thus far has been limited to a strictly potential flow solution, a comparison with experimental data from a blunt-based body is the only valid one that can be made, because the boundary layers on such bodies are usually so thin that they may be neglected. (For a discussion of this area, see the Introduction.) Unfortunately, the only blunt-based bodies for which complete sets of measured hydrodynamic coefficients exist are the Polaris missiles. The Al configuration was chosen for the comparison because it was hoped that the hydrodynamic coefficients for this early version of the missile could be declassified. This, however, proved impossible. Therefore, the comparison is made on the basis of the ratio of the coefficient determined from the theoretical calculations to the measured coefficient.

In calculating the hydrodynamic coefficients, it was necessary to account for the fact that on a blunt- or flat-based body the flow separates from the body at the base. For an observer traveling with an axisymmetric blunt-based body having (1) a large forward velocity, (2) a small angle of attack, and (3) small pitch rate, the flow behind the body travels rearward almost as though the body extended back for for some distance behind its true termination. Therefore, in computing the values of f_u, f_w, and f_q, a simple model which assumes the body to extend back indefinitely at its base diameter was used. Of course, in calculating the values of Z_w', Z_q', M_w', and M_q' from these values of f_u, f_w, and f_q, the integrations were performed over the true surface of the body only. In considering a model to use for calculating the values of h_w and h_q, it appeared likely that the pressure field created in the fluid due to a transverse or angular acceleration of the body would fairly strongly sense the presence of the abrupt termination of the body at the base. Hence the values of h_w and h_q used in calculating the values of Z_w', Z_q', M_w', M_q' were obtained with a model which assumes that the body ends at its true base termination, but does not have a solid surface across the base. Although several other models were tried for calculating values of f_u, f_w, and f_q and values of h_w and h_q, the two models discussed above, which are the simplest possible from a computational standpoint, were found to give the best results.

The question arises whether these representations of the base flow, which worked well for the A1 Polaris missile configuration, will work well for bodies of greatly different shape. Owing to a lack of experimental data, this question cannot be answered at this time and will have to await further experimental studies.

A comparison of the theoretical and measured hydrodynamic coefficients of the A1 Polaris missile is given in Table 1.

TABLE 1. Ratios of Theoretical (T) to Measured (M) Values of Hydrodynamic Coefficients, A1 Polaris Missile

Coefficient ratio	Numerical value of ratio	Error in theoretical coefficient
$\dfrac{(Z'_w)_T}{(Z'_w)_M}$	0.932	-6.8%
$\dfrac{(Z'_q)_T}{(Z'_q)_M}$	0.927	-7.3%
$\dfrac{(M'_w)_T}{(M'_w)_M}$	0.929	-7.1%
$\dfrac{(M'_q)_T}{(M'_q)_M}$	1.047	+4.7%
$\dfrac{(Z'_{\dot{w}})_T}{(Z'_{\dot{w}})_M}$	1.023	+2.3%
$\dfrac{(Z'_{\dot{q}})_T}{(Z'_{\dot{q}})_M}$	0.935	-6.5%
$\dfrac{(M'_{\dot{w}})_T}{(M'_{\dot{w}})_M}$	0.992	-0.8%
$\dfrac{(M'_{\dot{q}})_T}{(M'_{\dot{q}})_M}$	0.961	-3.9%

Values given by the new theoretical approach presented in this report are compared with the measured[4] values given in Ref. 8. As mentioned above, the comparison is made on the basis of the ratio of theoretical to measured values.

The largest error is 7.3%. Considering the probable experimental error in the measured hydrodynamic coefficients, this new theoretical approach appears to give values for blunt-based bodies that are adequate for most engineering work. Naturally, this supposition is tentative, since the comparison is limited to only one body shape.

A comparison similar to that shown in Table 1 was made using slender body theory. In Table 1, the mean of the absolute values of the percent error is 4.9; for slender body theory, it is 11.7. In Table 1, the maximum percent error is 7.3; for slender body theory, it is 23.5. It thus appears that the new theoretical approach yields a substantial improvement over slender body theory.

DISCUSSION AND CONCLUSIONS

A new method has been presented for calculating the hydrodynamic coefficients of axisymmetric bodies. This method utilizes an extended version of the Douglas potential flow computer program for axisymmetric bodies to determine those velocities and potentials at the body surface which are required to solve for the forces and moment acting on the body. The development here has been restricted to blunt- or flat-based bodies where the boundary layer on the body may be neglected. It is planned to extend this work to bodies having streamlined aftersection shapes where the thick boundary layer must be accounted for.

Based on a comparison with experiments limited to one body shape, it may tentatively be concluded that the new method yields hydrodynamic coefficient values for blunt-based bodies that are adequate for most engineering purposes. The successful application to a blunt-based body indicates that the method might eventually be extended to streamlined bodies.

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[4] The Z_w' and M_w' coefficients were measured in the water tunnel of the Ordnance Research Laboratory, Pennsylvania State University. The remaining coefficients were measured on the pitch-and-heave oscillator at the David Taylor Model Basin, Washington, D.C.

REFERENCES

1. U. S. Naval Ordnance Test Station, Inyokern. The Stability Coefficients of Standard Torpedoes, by M. R. Bottaccini. China Lake, Calif., NOTS, July 1954. (NAVORD Report 3346, NOTS 909.)

2. U. S. Naval Ordnance Test Station. Hydrodynamic Coefficients of Torpedo Bodies, by T. G. Lang, China Lake, Calif., NOTS, April 1955. (NAVORD Report 3485, NOTS 1107.)

3. ————. A Turbulent Boundary-Layer Calculation Method Based on the Law of the Wall and the Law of the Wake, by D. M. Nelson. China Lake, Calif., NOTS, November 1964. (NAVWEPS Report 8510, NOTS TP 3493.)

4. Douglas Aircraft Co. Calculation of Non-Circulatory Plane and Axially Symmetric Flows About or Within Arbitrary Boundaries, by A. M. O. Smith and Jesse Pierce. El Segundo, Calif., Douglas, April 1958. (Report No. ES 26988.)

5. ————. Calculation of Potential Flow About Bodies of Revolution Having Axes Perpendicular to the Free Stream Direction, by John L. Hess. El Segundo, Calif., Douglas, October 1960. (Report No. ES 29812.)

6. ————. Extension of the Douglas-Neumann Program for Axisymmetric Bodies to Include Calculation of Potential, Non-Uniform Cross Flow, Added Mass, and Conduction Problems, by John L. Hess. Long Beach, Calif., Douglas, September 1964. (Report No. 31765.)

7. Society of Naval Architects and Marine Engineers. Nomenclature for Treating the Motion of a Submerged Body Through a Fluid. New York, N. Y., SNAME, April 1950. (Technical and Research Bulletin No. 1-5.)

8. U. S. Naval Ordnance Test Station. Polaris Underwater Launch Program Investigations Progress Report No. 6, 1 July 1960 - 1 April 1961, Part 1: Analysis and Correlation of Results, by J. G. Rezin and D. M. Nelson. China Lake, Calif., NOTS, July 1961. (NAVWEPS Report 7765, NOTS TP 2743), CONFIDENTIAL.

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A new method is presented for calculating the hydrodynamic coefficients of axisymmetric bodies. The method utilizes an extended version of the Douglas potential flow computer program for axisymmetric bodies to calculate the potential and velocity distribution along the body required for the determination of the forces and moment acting on the body. Based on a comparison with experiment limited to one body shape, the method yields values of the hydrodynamic coefficients for blunt- or flat-based bodies which are adequate for most engineering work.

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